Probabilistic ‘If-Then’ Rules: On Bayesian Conditionals and Probabilistic Implications

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Abstract

Probabilistic rules “if A then B” rules are typically formalized as Bayesian conditionals $P(B|A)$, as many (e.g., Pearl) have argued that Bayesian conditionals are the correct way to think about such rules. However, there are challenges with standard inferences such as modus ponens, modus tollens, and rule chaining that might make probabilistic material implication a better candidate at times for rule-based systems employing forward-chaining; and arguably material implication is still suitable when information about prior or conditional probabilities is not available. We compare a probabilistic version of the material conditional from classical logic with Bayesian conditionals in the setting of interval-valued probability theory. We give quantitative treatments of familiar rules from logic to arrive at the best possible bounds on inferences in rule-based systems constituting directed acyclic graphs.

Introduction. The discussion of how to best render formally the intuitions behind “if-then” conditionals is still unresolved, in part because such conditionals can be used to capture analytical truths (“if x is an even number, x is divisible by two”), conceptual relationships (“if x is a human, then x is mortal”), inductive inferences (“if the sprinkler is on, the grass will become wet”), abductive inferences (“if the light switch is on but the light is off, the light bulb is broken”), normative constraints (“if the traffic light is red, you are not allowed to drive”), hypotheticals (“if I were to run fast, I would be out of breath”), and counterfactuals (“if the driver had been able to break in time, they would not have killed the deer”). From a logical point of view, the classical conditional $A \to B$ is false only when A is true and B is false, thus A and $A \to B$ cannot both be false at the same time, a fact that can be used to determine the consistency of a set of rules. Yet, treating conditionals as truth-functional has long been seen to be problematic (e.g., Adams 1965) and the widely accepted solution is to view such rules as probabilistic and best modeled by the Bayesian conditional $P(B|A)$ (e.g., Pearl 1988). At the same time, some have argued that under certain conditions the intuitive conditionals amounts to material implication (see Khoo and Mandelkern 2018, for a discussion) and that the Bayesian conditional is not appropriate for handling (indicative) conditionals. Moreover, some logically valid inferences such as the “hypothetical syllogism” $A \to B, B \to C \models A \to C$ (e.g., Hailperin 1984) are often probabilistically “uninformative” (e.g., Pfeifer and Kleiter 2006), as we will also show below.

The question then is how probabilistic versions of frequently used conditional inference schemes such as *modus ponens*, *modus tollens*, and *hypothetical syllogism* (used for chaining conditionals) compare to a probabilistic version of the material conditional. The goal of this paper is to shed light on the tradeoffs between the two interpretations of conditionals with respect to the various inference schemes. For this purpose, we use the standard embedding of propositional logic in probability theory, *probability logic* (e.g., Hailperin 1984), to compare probabilistic bounds on inferences with material conditionals vs. Bayesian conditionals and show that probabilistic material conditionals allow for better and faster inferences with a probabilistic conditional knowledge base. Moreover, we prove the best possible bounds on inferences in directed acyclic graphs induced by a system of probabilistic material conditionals.

References

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