Coherent lower and upper conditional previsions with respect to inner and outer Hausdorff measures and their insights for modelling human decisions in AI

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Abstract

A new mathematical model of coherent upper and lower conditional previsions defined by Hausdorff outer and inner measure is proposed to represent the conscious and unconscious activities of the human brain. The model of coherent upper and lower conditional previsions based on Hausdorff outer and inner measures allows to assess uncertainty measures according to the complexity of the conditioning event that represent the given information. The two non-linear functionals represent different binary relations between random variables since preference orderings described in terms of coherent lower conditional previsions satisfy the antisymmetric property which is not satisfied by the binary relation represented by their conjugate coherent upper conditional previsions. The two binary relations can describe the activity of the conscious human thought ruled by the antisymmetric property and of the unconscious human thought which is governed by the symmetric principle and the generalization principle according to the theory developed by Matte-Blanco. In the Prospect Theory human reasoning and human decision making is investigated by different experiments which evidence biases of human intuition. Experimental methods lead to describe the dual process of the brain activity as regulated by two different ways of thinking namely fast and slow thinking denoted also as System 1 and System 2, respectively. System 1 regulates intuitive, involuntary, unconscious and effortless activities while System 2 is the conscious part of the brain in charge of logical reasoning. Accordingly, a mathematical argument is developed to show how a rational choice model based on coherent lower and upper conditional previsions defined by Hausdorff outer and inner measures can be useful in the following two cases. In the case 1, an alternative mathematical solution based on the model features is concisely depicted and applied to review the original KT Lindas experiment. The so called conjunction fallacy is solved by adopting two non-additive conditional probability (i.e., coherent upper and lower conditional probabilities), instead of the single unconditional probability of probabilistic reasoning. The model of coherent upper and lower conditional probabilities based on Hausdorff outer and inner measures allows to assess uncertainty measures according to the complexity of the conditioning event that represent the given information. Here, the two non-linear functionals represent different binary relations between random variables since preference orderings, described in terms of coherent lower conditional previsions, satisfy the antisymmetric property which is not satisfied by the binary relation represented by their conjugate coherent upper conditional previsions. In the case 2, as for Linda's solution, the model features a reapplied to explain mathematically the bias of selective attention described in the so-called invisible gorilla experiment (Simons 1999), that is often taken as a characteristic example of the inescapable limitations of human perception. In a nutshell, once people are concentrated on doing a specific action, they do not notice unexpected events (having 0 probability) occurring in the meantime. When applying the model, selective attention is no longer a bias since it is able to explain this function of the human brain mathematically and without incoherencies.