Joint Desirability Foundations of Social Choice and Opinion Pooling

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Abstract

In this work we develop joint foundations for the fields of *social choice* and *probabilistic opinion pooling* using coherent sets of desirable gambles, a general uncertainty model that allows to encompass both complete and incomplete preferences as well as probabilities or sets of probabilities (Williams 1975; Walley 1991; Augustin et al. 2014).

On the one hand indeed, the research field of social choice theory (Feldman and Serrano 2006; Sen 1986) examines the question of aggregating beliefs and values of a number of rational agents expressed through preference relations over alternatives. Its aim is to define social functions that best represent the preferences of the voters. As such, it is not directly concerned with questions of probability. On the other hand, instead, the aim of the related field of probabilistic opinion pooling is finding a model that best 'summarises' a given number of probabilistic beliefs (Lindley, Tversky, and Brown 1979).

Although social choice and opinion pooling have a number of distinct features, they can also be gathered together under the umbrella of *beliefs aggregation*. Both indeed regard the aggregation of the belief models of a number of voters or experts, being in the shape of preferences over a number of alternatives (in social choice) or uncertainty models about some experiment (in opinion pooling). It may be useful therefore to give them a common treatment that can later be particularised to each problem to deal with its specificities.

To accomplish this aim, we reformulate their founding elements using coherent sets of gambles, a general belief model that can include as particular cases (possibly incomplete) preference relations as well as (sets of) probabilities. It has its roots in the behavioural theory of imprecise probabilities, which was later much developed by Walley (1991). It is sufficiently general while at the same time easier to work with than sets of probabilities. In addition, sets of desirable gambles permit us to simultaneously deal with considerations of beliefs and values (Zaffalon and Miranda 2017) and to consider any domain and possibility space (Zaffalon and Miranda 2018). They can also deal more effectively than other models with the problem of sets of measure zero (Miranda and Zaffalon 2010).

This joint view gives us the possibility to provide new perspective of traditional results of social choice such as the celebrated *Arrow's theorem* (Arrow 1950), which establishes limits to what is possible to do in order to satisfy some reasonable properties for a voting system while avoiding dictatorial solutions. In particular, in this context, the possibility given by coherent sets of gambles of easily making judgements of incompletness and incomparability offers the key to escape from the impossibility of Arrow's result.

We analyze also traditional extensions of this result (Jain 2015; Weymark 1984), providing sufficient conditions for the existence of an oligarchy and a democracy. In particular, we show that the only possible way to obtain a democracy in this context is by avoiding conflicts among individuals, endorsing only those opinions that are shared by all subjects: stated differently, incompleteness is necessary for democracy.

We use then the same framework to analyse opinion pooling. In this context, we argue that weak Pareto (unanimity) should be given the status of a rationality requirement and we use this to discuss the aggregation of experts' opinions based on probability and utility.

Our unified approach permits us also to investigate whether some axioms and results from one of the fields are sensible in the other. In this respect, for example, we show how some of the desirable criteria in Walley (1982) for an aggregation rule are respected by our vision. Another advantage is that, taking into account the modelling of decision making using the theory of sets of desirable gambles in Zaffalon and Miranda (2017, 2018), it should be a small step from this paper to make a similar modelling of multicriteria decision making, where criteria take the role of the voters in our reformulation of social choice. Some comments in this direction can be found in Dubois, Fargier, and Perny (2002).

We conclude reformulating our main results also in terms of sets of probabilities or *coherent lower previsions* (lower expectation functionals) and discussing in some details some earlier works connected to our paper.

As future lines of research, we would like to tighten even more the relation between our work and traditional social choice in order to provide a full probabilistic treatment of this research field as well as extend our work to an infinite number of experts, or voters. Another interesting area of investigation could be to try to move our results to computational social choice (Chevaleyre et al. 2007; Rossi, Venable, and Walsh 2011), a research area that focuses on the algorithmic tasks in social choice and their complexity. In this context, it could be interesting to possibly modifying our reformulation of social choice taking into account limitations of computational resources directly at the level of the axioms of desirability.

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