Title: Parameterized Analysis for the Group Activity Selection Problem on Graphs

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Abstract: In varied real-life situations, ranging from carpooling to workload delegation, several activities are to be performed, to which end each activity is assigned to a group of agents. These situations are captured by the Group Activity Selection Problem (GASP). Notably, relevant relations among agents, such as acquaintanceship or physical distance, can often be modeled naturally using graphs. To exploit this modeling ability, Igarashi, Peters and Elkind [AAAI 17] introduced gGASP.Specifically, it is required that each group corresponds to a connected set of the underlying graph. In addition, to enforce the execution of the activities in practice, no individual desires to desert its group in favor of joining another group. In other words, the assignment is Nash stable. In this paper, we study gGASP with Nash stability (Nash Stable gGASP), whose objective is to compute such an assignment. This problem is computationally hard even on such restricted topologies as paths and stars, which naturally led Igarashi, Bredereck, Peters and Elkind [AAAI 17, AAMAS 17] to the study Nash Stable gGASP in the framework of parameterized complexity. We take this line of investigation forward, significantly advancing the state-of-the-art. First, we show that Nash Stable gGASP is NP-hard even when merely one activity is present. In fact, this special case remains NP-hard when we further restrict the graph to have maximum degree \Delta =5. Consequently, Nash Stable gGASP is not fixed-parameter tractable (FPT), or even XP, when parameterized by p + \Delta, where p is the number of activities. However, we are able to design a parameterized algorithm for Nash Stable gGASP on general graphs with respect to p+\Delta+t, where t is the maximum size of a group. Finally, we develop an algorithm that solves Nash Stable gGASP on graphs of bounded treewidth tw in time $4^p \sum(n+p)^{\{O(tw)\}}$. Here, \Delta+t can be arbitrarily large. Along the way, we resolve several open questions regarding Nash Stable gGASP.